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Comments on the Bagger-Lambert theory and multiple M2-branes

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Abstract: We study the SO(8) superconformal theory proposed recently by Bagger and Lambert as a possible worldvolume theory for multiple M2-branes. For their explicit example with gauge group SO(4), we rewrite the theory (originally formulated in terms of a three-algebra) as an ordinary $\mathrm{SU}(2) \times \mathrm{SU}(2)$ gauge theory with bifundamental matter. In this description, the parity invariance of the theory, required for a proper description of M2-branes, is clarified. We describe the subspace of scalar field configurations on which the potential vanishes, correcting an earlier claim. Finally, we point out, for general three-algebras, a difficulty in constructing the required set of superconformal primary operators which should be present in the correct theory describing multiple M2-branes.

Keywords: D-branes, M-Theory.

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Contents

1. Introduction

In this note, we investigate a fascinating 2+1 dimensional field theory proposed recently by Bagger and Lambert [1 − 3] as a worldvolume description of multiple M2-branes in M-theory.[1] Like the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory in 3+1 dimensions, this theory has an explicit Lagrangian description, which the authors construct based on a new algebraic structure called a "three-algebra," (an equivalent structure was proposed in [5]) reviewed in section 2 below. Bagger and Lambert have shown that one obtains an $\mathcal{N} = 8$ supersymmetric theory with manifest SO(8) R-symmetry given any such three-algebra, and argued that the theory must be superconformally invariant. In the original work by Bagger and Lambert, only a single example of such an algebra was given [2].

In this note, we explicitly rewrite the Bagger-Lambert theory in this example as an ordinary gauge theory with gauge group SU(2) × SU(2) and matter in the bifundamental representation. This construction clarifies how the theory is able to maintain parity invariance - required if the theory is to describe M2-branes - despite the presence of a Chern-Simons term. Specifically, we find that twisted Chern-Simons term in the original formulation of the theory breaks up into separate Chern-Simons terms for the two SU(2) gauge fields with opposite sign. While each of these is odd under parity, the combination is parity-invariant if we stipulate an exchange of the two gauge fields under parity.

We next analyze the scalar potential of the theory, and characterize the space of configurations for which this potential vanishes. For the explicit example, we find that the space of gauge-inequivalent scalar field configurations on which the potential vanishes is $(R^8 \times R^8)/O(2)$ where the $O(2)$ rotates the two R^8 factors into each other. We show

[1] For a review of properties of M2-branes, see [6].

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that the $\mathrm{SU}(2)\times\mathrm{SU}(2)$ gauge symmetry is broken to $\mathrm{U}(1)$ at a generic point on this space, while there is a special vector subspace that preserves $\mathrm{SU}(2)$.

Finally, we comment on gauge-invariant operators in the Bagger-Lambert theory.[2] AdS/CFT duality predicts that these theories should contain superconformal primary operators in traceless symmetric representations of SO(8) with any number of indices (with the exception of the theory of two M2-branes, where due to the stringy exclusion principle, such representations with an odd number of indices are absent from the interacting part of the theory). On the other hand, we argue that the odd-index representations cannot be constructed from fields in the Bagger-Lambert theory, unless there is some additional algebraic structure (e.g. an ordinary product).

Note: After this work was completed, the papers [10 − 12] appeared, which have some overlap with the present work.

2. Review of the Bagger-Lambert construction

To begin, we briefly recall the Bagger-Lambert construction of a class of SO(8) superconformal theories. This starts by defining a three-algebra to be a vector space with inner product, together with a completely antisymmetric triple product, where the inner product and the triple product are defined by their action on a basis T^a by

$$\mathrm{Tr}(T^aT^b) = h^{ab}$$

and

$$[T^a, T^b, T^c] = f^{abc}{}_d T^d \ .$$

The triple product is required to satisfy

$$[A, B, [C, D, E]] = [[A, B, C], D, E] + [C, [A, B, D], E] + [C, D, [A, B, E]] \ ,$$

so that the operation $[A, B, *]$ behaves like a derivation when acting on a triple product of elements, and also

$$\mathrm{Tr}(A[B, C, D]) = -\mathrm{Tr}([A, B, C], D)$$

so that $f^{abcd} = h^{de} f^{abc}{}_e$ must be totally antisymmetric.

Given such an algebraic structure, one constructs a field theory starting with eight algebra-valued scalars X^I_a transforming in the vector of SO(8), eight algebra valued spinors transforming in the antichiral spinor representation of SO(8), and a gauge field $A_{\mu ab}$ antisymmetric in the algebra indices. The spinors may be arranged into a single 32-component Weyl spinor Ψ_a, obeying

$$\Gamma_{012}\Psi = -\Psi \ ,$$

where we will use the notation Γ^I to denote 32×32 Dirac matrices. Defining

$$\tilde{A}^c_{\mu d} = f^{abc}{}_d A_{\mu ab}$$

[2] This section of the paper arose from a discussion with Jaume Gomis, who suggested thinking about chiral operators in the Bagger-Lambert theory.

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we can define covariant derivatives $(\tilde{D}_\mu X^I)_a$ and $(\tilde{D}_\mu \Psi)_a$ and a field strength $\tilde{F}^a_{\mu\nu b}$ via the standard definitions, it may be checked that these transform covariantly under a gauge-symmetry

$$\begin{aligned}
\delta X^I_c &= \tilde{\Lambda}^d{}_c X^I_d \\
\delta \Psi_c &= \tilde{\Lambda}^d{}_c \Psi_d \\
\delta \tilde{A}^d_{\mu c} &= \tilde{D}_\mu \tilde{\Lambda}^d{}_c \\
\tilde{\Lambda}^c{}_d &\equiv f^{abc}{}_d \Lambda_{ab} \ .
\end{aligned}$$

With these definitions, the Bagger-Lambert action is

$$\begin{aligned}
\mathcal{L} = &-\frac{1}{2} D^\mu X^{Ia} D_\mu X^I_a + \frac{i}{2} \bar{\Psi}^a \Gamma^\mu D_\mu \Psi_a + \frac{i}{4} \bar{\Psi}_b \Gamma_{IJ} X^I_c X^J_d \Psi_a f^{abcd} \\
&-\frac{1}{12} \mathrm{Tr}([X^I, X^J, X^K][X^I, X^J, X^K]) \\
&+\frac{1}{2} \epsilon^{\mu\nu\lambda} (f^{abcd} A_{\mu ab} \partial_\nu A_{\lambda cd} + \frac{2}{3} f^{cda}{}_g f^{efgb} A_{\mu ab} A_{\nu cd} A_{\lambda ef})
\end{aligned}$$

In [2], this was shown to be invariant under gauge transformations and 16 supersymmetries:

$$\begin{aligned}
\delta X^I_a &= i\bar{\epsilon} \Gamma^I \Psi_a \\
\delta \Psi_a &= D_\mu X^I_a \Gamma^\mu \Gamma^I \epsilon - \frac{1}{6} X^I_b X^J_c X^K_d f^{bcd}{}_a \Gamma^{IJK} \epsilon \\
\delta \tilde{A}^b_{\mu a} &= i\bar{\epsilon} \Gamma_\mu \Gamma_I X^I_c \Psi_d f^{cdb}{}_a \ .
\end{aligned}$$

where the spinor ϵ has the opposite chirality from Ψ,

$$\Gamma_{012} \epsilon = \epsilon \ .$$

2.1 Example

An example of this algebraic structure was given by Bagger and Lambert in [2]. In this case, the vector space is R^4 and we can take

$$\begin{aligned}
h^{ab} &= \delta^{ab} \\
f^{abcd} &= f \epsilon^{abcd}
\end{aligned}$$

for some constant f. In this case, the triple product is the natural generalization to four dimensions of the usual cross product: it gives a new vector perpendicular to the vectors in the product whose length is the signed volume of the parallelepiped spanned by the vectors.

3. Description as a bifundamental gauge theory

We will now see that for the known case just described, the Bagger-Lambert theory may be rewritten explicitly as an ordinary gauge theory with gauge group as SU(2) × SU(2), and matter in the bifundamental representation.

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Under the SU(2) $\times$ SU(2) decomposition, a real vector of SO(4) becomes a bifundamental of SU(2) $\times$ SU(2), obeying the reality condition

$$X_{\alpha\dot{\beta}} = \epsilon_{\alpha\beta}\epsilon_{\dot{\beta}\dot{\alpha}}(X^\dagger)^{\dot{\alpha}\beta}$$

Explicitly, we can write

$$X^I = \frac{1}{2}\begin{pmatrix} x_4^I + ix_3^I & x_2^I + ix_1^I \\ -x_2^I + ix_1^I & x_4^I - ix_3^I \end{pmatrix}$$

with a similar expression for the spinor.

The gauge field $A_{\mu ab}$ may be decomposed into self-dual and anti-self-dual parts

$$A_{\mu ab} = -\frac{1}{2f}(A^+_{\mu ab} + A^-_{\mu ab}) \qquad\qquad A^\pm_{\mu ab} = \pm\frac{1}{2}\epsilon_{abcd}A^\pm_{\mu cd}$$

in terms of which we define

$$A_\mu = A^+_{\mu 4i}\sigma_i \qquad\qquad \hat{A}_\mu = A^-_{\mu 4i}\sigma_i$$

where the Pauli matrices σ_i are normalized so that $\mathrm{Tr}(\sigma_i\sigma_j) = 2\delta_{ij}$. Making all the replacements, we find that the action becomes

$$\begin{aligned}\mathcal{L} = {}& \mathrm{Tr}(-(D^\mu X^I)^\dagger D_\mu X^I + i\bar{\Psi}^\dagger\Gamma^\mu D_\mu\Psi) \\ & +\mathrm{Tr}\Big(-\frac{2}{3}if\bar{\Psi}^\dagger\Gamma_{IJ}(X^IX^{J\dagger}\Psi + X^J\Psi^\dagger X^I + \Psi X^{I\dagger}X^J) \\ & \qquad\qquad\qquad\qquad -\frac{8}{3}f^2X^{[I}X^{J\dagger}X^{K]}X^{K\dagger}X^JX^{I\dagger}\Big) \\ & +\frac{1}{2f}\epsilon^{\mu\nu\lambda}\mathrm{Tr}(A_\mu\partial_\nu A_\lambda + \frac{2}{3}iA_\mu A_\nu A_\lambda) - \frac{1}{2f}\epsilon^{\mu\nu\lambda}\mathrm{Tr}\Big(\hat{A}_\mu\partial_\nu\hat{A}_\lambda + \frac{2}{3}i\hat{A}_\mu\hat{A}_\nu\hat{A}_\lambda\Big)\end{aligned}$$

where

$$D_\mu X^I = \partial_\mu X^I + iA_\mu X^I - iX^I\hat{A}_\mu$$

The supersymmetry transformation rules above become

$$\begin{aligned}\delta X^I &= i\bar{\epsilon}\Gamma^I\Psi \\ \delta\Psi &= D_\mu X^I\Gamma^\mu\Gamma^I\epsilon + \frac{2}{3}fX^IX^{J\dagger}X^K\Gamma^{IJK}\epsilon \\ \delta A_\mu &= f\bar{\epsilon}\Gamma_\mu\Gamma_I(X^I\Psi^\dagger - \Psi X^{I\dagger}) \\ \delta\hat{A}_\mu &= f\bar{\epsilon}\Gamma_\mu\Gamma_I(\Psi^\dagger X^I - X^{I\dagger}\Psi)\,.\end{aligned}$$

Note that the twisted Chern-Simons term in the original formulation has decomposed into two separate ordinary Chern-Simons terms for A and $\hat{A}$, albeit with opposite signs. The usual constraint that arises by demanding invariance under large gauge transformations then requires us to choose

$$f = \frac{2\pi}{k}$$

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where the level k is an integer. It is particularly interesting to note that after a rescaling $A \to \sqrt{f}A$, all interaction terms in the theory are proportional to positive powers of f, so the theory becomes weakly coupled in the limit of large k. Thus, the theory can be solved exactly in the limit of large level and studied in perturbation theory for large level.

3.1 Parity invariance

In the explicit expression for the action above it is straightforward to see how the theory manages to have a parity invariance symmetry despite the presence of Chern-Simons terms which are parity-odd.[3] Since the Chern-Simons terms for A and $\hat{A}$ have opposite sign, the action of parity combined with the switch

$$A_\mu \leftrightarrow \hat{A}_\mu$$

leaves the gauge field part of the action invariant. In order to make the remainder of the bosonic action parity invariant, we must also demand a transformation

$$X^I \leftrightarrow X^{I\dagger} .$$

Invariance of the full action presumably now follows from supersymmetry, however we refer the reader to [10] for a more explicit discussion of parity for the fermionic terms in the action. Inspection of the kinetic term for the fermions shows that the correct transformation for these is

$$\Psi \leftrightarrow \Gamma^1 \Psi^\dagger .$$

where the Γ^1 factor comes from the standard parity transformation. All of these parity transformations may be seen to arise in the original language from a transformation that combines spacetime parity and a flip of the (234) directions in the internal space.

4. Scalar potential

In this section, we consider the scalar potential for the SO(4) example of the Bagger-Lambert theory, and characterize the set of gauge-inequivalent scalar field configurations for which the potential vanishes.[4]

To begin, we recall that in this case, the bosonic matter fields are 8 (distinguishable) vectors in an R^4 that is rotated by the gauge symmetry. The triple product gives a new vector perpendicular to the vectors in the product whose length is the signed volume of the parallelepiped spanned by the vectors. The bosonic potential is proportional the square of this volume, summed over each possible triple of vectors.

With this description, it is clear that the bosonic potential vanishes if and only if any three of the vectors lie in the same plane. This space is labeled by ordered sets of 8 vectors all of which lie in the same plane, with sets related by overall rotations in R^4 considered equivalent. Without loss of generality, we may assume that all vectors lie in the $x_3 - x_4$

[3]Here, we take parity to be defined as a reflection in the x_1 direction.

[4]This was considered previously in [3] but we find a slightly different result.

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plane; the 8 x_3 coordinates and 8 x_4 coordinates form ordered octuplets which are rotated into each other by the residual $O(2)$ gauge symmetry. We conclude that the space of gauge-inequivalent scalar field configurations for which the potential vanishes in the SO(4) Bagger-Lambert theory is $(R^8 \times R^8)/O(2)$.[5]

We briefly comment on the symmetry-breaking structure for the scalar field configurations just described. In our bifundamental notation, these configurations with vanishing potential are exactly the set of matrices X^I that are diagonal up to gauge transformations. A generic such point may be described by a matrix

$$X^I = \begin{pmatrix} z^I & 0 \\ 0 & \bar{z}^I \end{pmatrix}$$

where z^I are complex. This preserves residual U(1) gauge symmetry, generated by $A_\mu = \hat{A}_\mu \propto \sigma_3$. On the vector subspace where z^I is real, a full SU(2) is preserved, generated by $A_\mu = \hat{A}_\mu$.

5. Superconformal operators

The correct superconformal theory describing multiple M2-branes is believed to be dual to M-theory on $AdS^4 \times S^7$, with the curvature of the spacetime in Planck units determined by the number N of M2-branes [13]. For large N, the curvature is small, and supergravity should provide a good description of the low energy physics. Thus, low-dimension operators in the superconformal field theory should be in one-to-one correspondence with the spectrum of supergravity fluctuations around the $AdS^4 \times S^7$ background. The single-particle states were determined in [14] and shown in [15 − 17] to correspond to a single series of irreducible representations of the superconformal algebra, labeled by an integer $k \geq 1$. The operators of lowest dimension in each of these representations, are superconformal primary operators of dimension $k/2$ transforming in the symmetric traceless k-index representation of the SO(8) R-symmetry group. Thus, such operators should be present in the conformal field theory that describes the decoupled physics of a large number of M2-branes.

In the Bagger-Lambert theory for a general 3-algebra, the matter fields X^I_a and $(\Psi_\alpha)_a$ transform in the 8_v and 8_c representations of SO(8) and carry a single algebra index (they are elements of the algebra itself). Meanwhile, the gauge fields are SO(8) invariant and carry two algebra indices. To form gauge invariant operators, all algebra indices must be contracted. In the absence of any additional algebraic structure, the only invariant tensors that we have to work with are h^{ab} and f^{abcd} (this is certainly true in the SO(4) example). As a result, all gauge-invariant operators must have an even number of matter fields. The tensor product of two 8_c representations gives representations appearing in the tensor product of even numbers of 8_v representations. Thus, the SO(8) representation of any bosonic gauge-invariant operator must be an ordinary tensor representation of SO(8) with an even number of indices. In particular, it seems impossible to construct the expected operators in symmetric, traceless representations of SO(8) with an odd number of indices.

[5]Our result differs from the one in [3] by the presence of the $O(2)$ factor.

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For the special case of two M2-branes, the situation is slightly better, due to the stringy exclusion principle [21, 18 − 20], which reduces the expected spectrum of operators from the full supergravity result. In this case, the expected result for the operator spectrum (conjectured in ([22])) for the interacting part of the M2-brane theory does not contain symmetric traceless representation with an odd number of SO(8) indices.[6]

Our discussion above shows that if the Bagger-Lambert theory based on some example of a 3-algebra is to describe the worldvolume theory of three or more M2-branes, there must be some additional algebraic structure that allows us to form gauge-invariant operators more general than those constructed from the invariants h^{ab} and f^{abcd} alone.

Note added. The journal version of this paper has been updated to better reflect the current state of knowledge about the Bagger-Lambert theory and its relation to M2-branes. When the original version of this paper was written, the existing proposal was that the SO(4) example of the Bagger-Lambert theory (plus a decoupled free sector) described the physics of 3 M2-branes in uncompactified M-theory. The discussion in the original sections 4 and 5 was designed in part to call into question this proposal. More recently, a new explicit proposal for the M-theory interpretation of the SO(4) Bagger-Lambert theory has been put forth [7 − 9], namely, that this theory (without the decoupled sector) describes the physics of two M2-branes on a certain type of orbifold (whose nature depends on the level parameter of the Chern-Simons theory).

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